



MA480 Topology and Geometry (Online)

Instructor Information	<p style="text-align: center;">Wei Zhao Home Institution: East China University of Science and Technology Email: wzhao@ecust.edu.cn Office Hours: To be determined</p>		
Term	<p style="text-align: center;">June 27, 2022 - July 22, 2022</p>	Credits	<p style="text-align: center;">4 units</p>
Course Delivery	<p style="text-align: center;">The class will be delivered online. Besides recorded lecture videos, the instructor will arrange 2.5 hours' real-time interactions with students for every week (via VooV Meeting). The workload expected to pass this course is at least 15 hours per week.</p>		
Required Texts (with ISBN)	<p style="text-align: center;">Glen E. Bredon, Topology and Geometry, (Graduate Texts in Mathematics 139) ISBN 10: 0387979263. ISBN 13: 9780387979267</p> <p style="text-align: center;">Dmitri Burago, Yuri Burago, Segei Ivanov, A Course in Metric Geometry, (Graduate Studies in Mathematics 33) ISBN 10: 0821821296. ISBN 13: 978-0821821299</p> <p style="text-align: center;">Frank W. Warner, Foundations of Differentiable Manifolds and Lie Groups, (Graduate Texts in Mathematics 94) ISBN 10: 0387908943. ISBN 13: 978-0387908946</p>		
Prerequisite	<p style="text-align: center;">Basic Calculus (Convergence of sequences, Continuity of functions, Smoothness of functions)</p>		



Course Overview

The purpose of this course is to expand some of basic topological and geometric ideas used in studying constructions and properties of spaces. The first part is mainly focused on general topology (i.e., point-set topology), in which functions, subspaces, connectivity, separation, compactness of a topological space are introduced and investigated. The second part is devoted to the investigation of metric spaces. As a special and important example, the spaces endowed with the Gromov-Hausdorff distance are discussed. The last part is about differential manifolds, in which some basic concepts are introduced.

Learning Outcomes

LO1. Demonstrate knowledge of the definitions of metric spaces and topological spaces, and the relationships between them.

LO2. Demonstrate knowledge of the basic properties of functions on topological spaces, and the definition of subspaces.

LO3. Classify topological spaces by separation axioms; demonstrate knowledge of the definition and basic properties of Hausdorff spaces.

LO4. Demonstrate knowledge of the definition of compact spaces; understand the influence of compactness assumption to topological properties.

LO5. Understand the construction of product spaces; construct simple proofs of propositions dealing with product spaces.

LO6. Understand the construction of length spaces and the relationship between metric spaces and induced length spaces.

LO7. Demonstrate knowledge of the definition and basic properties of the Gromov-Hausdorff distance.

LO8. Demonstrate knowledge of the definition of manifolds; construct some simple examples of manifolds.

LO9. Understand the geometric meaning of tangent vectors and cotangent convectors; deal with simple calculations on differential manifolds.

Assignments

Assignments are written work, which are given in the recorded video lectures. Most of them are the problems from the reference books “Topology and Geometry” and “A Course in Metric Geometry”.

- Assignments of Day 1, Day 2 and Day 3 are due on Day 6;
- Assignments of Day 6, Day 8 and Day 9 are due on Day 11;
- Assignments of Day 11, Day 13 and Day 14 are due on Day 16.

Assessment Type

Final exam	Outcomes assessed: LO1, LO2, LO3, LO4, LO5, LO6, LO7, LO8, LO9
Assignments	Outcomes assessed: LO1, LO2, LO3, LO4, LO5, LO6, LO7, LO8, LO9
Quiz1	Outcomes assessed: LO1, LO2, LO3, LO4
Quiz2	Outcomes assessed: LO5, LO6, LO7, LO8, LO9



Grading Policy

Final exam	40%
Quizzes	20%
Assignments	30%
Attendance	10%

Grading Scale is as follows

Number grade	Letter grade	GPA
90-100	A	4.0
85-89	A-	3.7
80-84	B+	3.3
75-79	B	3.0
70-74	B-	2.7
67-69	C+	2.3
65-66	C	2.0
62-64	C-	1.7
60-61	D	1.0
≤59	F (Failure)	0



Class Schedule

Date	Lecture	Readings	Online Teaching Arrangement
Day 1	Basic concepts of metric spaces; Introduction to topological spaces.	§I.1 of “Topology and Geometry”; §1.1-1.3 of “A Course in Metric Geometry”	Approximately 90-minute pre-recorded video lectures
Day 2	Functions on a topological space; Introduction to subspaces of a topological space.	§I.2-I.3 of “Topology and Geometry”; §1.4 of “A Course in Metric Geometry”	Approximately 90-minute pre-recorded video lectures
Day 3	Introduction to connectivity and components; Examples.	§I.4 of “Topology and Geometry”;	Approximately 90-minute pre-recorded video lectures
Day 4	Review and Exercise: How to solve basic topological/geometric problems (I).	§I.1- I.4 of “Topology and Geometry”; §1.1-1.4 of “A Course in Metric Geometry”	Approximately 90-minute pre-recorded video lectures plus 75-minute online interaction via VooV Meeting
Day 5	Review and Exercise: How to solve basic topological/geometric problems (II)	§I.1- I.4 of “Topology and Geometry” §1.1-1.4 of “A Course in Metric Geometry”	Approximately 90-minute pre-recorded video lectures plus 75-minute online interaction via VooV Meeting
Day 6	Introduction to separation axioms; Properties/Examples of Hausdorff spaces.	§I.5 of “Topology and Geometry”	Approximately 90-minute pre-recorded video lectures
Day 7	Introduction to compactness; Solutions to homework of last week.	§I.7 of “Topology and Geometry”;	Approximately 90-minute pre-recorded video lectures plus 75-minute online interaction via VooV Meeting
Day 8	Properties of compact spaces (I)	§I.7, I.9 of “Topology and Geometry”; §1.5-1.6 of “A Course in Metric Geometry”	Approximately 90-minute pre-recorded video lectures
Day 9	Properties of compact spaces (II) Review and Exercises: How to solve topological/geometric problems concerned with compactness.	§I.7, I.9 of “Topology and Geometry”; §1.5-1.6 of “A Course in Metric Geometry”	Approximately 90-minute pre-recorded video lectures plus 75-minute online interaction via VooV Meeting
Day 10	Quiz 1	Contents from Day 1 to Day 9	



Day 11	Introduction to product spaces; Properties of product spaces; Examples	§I.8 of “Topology and Geometry”	Approximately 90-minute pre-recorded video lectures
Day 12	Introduction to length spaces, an example of metric spaces; Solutions to homework of last week.	§2.1-2.3 of “A Course in Metric Geometry”	Approximately 90-minute pre-recorded video lectures plus 75-minute online interaction via VooV Meeting
Day 13	Gromov-Hausdorff distance	§7.1-7.3 of “A Course in Metric Geometry”	Approximately 90-minute pre-recorded video lectures
Day 14	Convergence under Gromov-Hausdorff topology	§7.4-7.5 of “A Course in Metric Geometry”	Approximately 90-minute pre-recorded video lectures
Day 15	Review and Exercises: How to deal with the problems concerned with product/metric/length spaces.	§I.8 of “Topology and Geometry” §2.1-2.3, 7.1-7.5 of “A Course in Metric Geometry”	Approximately 90-minute pre-recorded video lectures plus 75-minute online interaction via VooV Meeting
Day 16	Introduction to manifolds Examples	§2.2-2.3 of “Topology and Geometry” Pages 1-8 of “Foundations of Differentiable Manifolds and Lie Groups”	Approximately 90-minute pre-recorded video lectures
Day 17	Tangent/cotangent vectors; Solutions to homework of last week.	§2.5 of “Topology and Geometry” Pages 11-19 of “Foundations of Differentiable Manifolds and Lie Groups”	Approximately 90-minute pre-recorded video lectures plus 75-minute online interaction via VooV Meeting
Day 18	Vector fields and flows; Tangent bundles.	§2.8-2.9 of “Topology and Geometry” Pages 19-20, 34-37 of “Foundations of Differentiable Manifolds and Lie Groups”	Approximately 90-minute pre-recorded video lectures
Day 19	Review and Exercises: How to deal with the problems concerned with manifolds	§2.2-2.3, 2.5, 2.8-2.9 of “Topology and Geometry” Pages 1-8, 11-20, 34-41 of “Foundations of Differentiable Manifolds and Lie Groups”	Approximately 90-minute pre-recorded video lectures plus 75-minute online interaction via VooV Meeting
Day 20	Quiz 2	Contents from Day 11 to Day 19	