



## Shanghai Jiao Tong University

### MA197 Algebra and Number Theory (Online)

<b>Instructor Information</b>	Wanchunzi Yu Home Institution: Bridgewater State University Email: wyu@bridgew.edu		
<b>Term</b>	December 17, 2020 - January 8, 2021	<b>Credits</b>	4 units
<b>Course Delivery</b>	The course will be delivered in the format of online. Other than recorded lecture videos, the instructor will arrange 3 hours' real-time interaction with students per week (via zoom meeting). The workload students are expected to complete to properly pass this course is about 12-16 hours per week. Exams are proctored under zoom-meeting camera.		
<b>Required Texts (with ISBN)</b>	A Book of Abstract Algebra, 2 <sup>nd</sup> edition, by Charles C Pinter. ISBN-13: 978-0486474175 ISBN-10: 0486474178 Elements of Number Theory, by John Stillwell ISBN-13: 978-1441930668 ISBN-10: 1441930663		
<b>Prerequisite</b>	N/A		



## Course Overview

This course introduces students to a rigorous study of algebra and number theory. During the first part of the course we will study natural numbers and integers, the Euclidean algorithm, congruence arithmetic, the RSA cryptosystem, quadratic integers, and quadratic reciprocity from the textbook of the Elements of Number Theory. We will learn operations, groups, subgroups, examples of groups and subgroups, as well as quotient groups from a book of abstract algebra.

## Learning Outcomes

Upon successful completion of this course, students will be conversant with

- understanding concepts of numbers and integers
- the definitions, examples, and applications of groups and subgroups
- developing problem solving skills
- apply concepts and theories to solve problems
- improvements of mathematical thinking skills

## Course Structure

1. Asynchronous Hours: Sunday through Thursday, total 25 hours Pre-recorded videos will be posted on SJTU SCE online learning platform.
2. Synchronous Hours: Beijing Time: Monday: 7:30 – 9 pm, Thursday: 8:30 – 10 am



### Grading Policy

Three Assignments	30%
Quizzes/Attendance	20%
Middle Exam	25%
Final Exam	25%

### Grading Scale is as follows

Number grade	Letter grade	GPA
90-100	A	4.0
85-89	A-	3.7
80-84	B+	3.3
75-79	B	3.0
70-74	B-	2.7
67-69	C+	2.3
65-66	C	2.0
62-64	C-	1.7
60-61	D	1.0
≤59	F (Failure)	0



**Class Schedule**

Date	Lecture	Arrangement	Readings
Day 1	1.1 Natural Numbers 1.2 Induction 1.3 Integers 1.4 Division with remainder 1.5 Binary notation 1.6 Diophantine equations	1.5 hours Zoom meeting 0.75 hours pre-recorded video	Elements of Number Theory Chapter 1
Day 2	1.7 The Diophantus chord method 1.8 Gaussian integers 2.1 The gcd by subtraction 2.2 The gcd by division with remainder 2.3 Linear representation of the gcd 2.4 Primes and factorization	2.25 hours pre-recorded video	Elements of Number Theory Chapter 1 & Chapter 2
Day 3	2.5 Consequences of unique prime factorization 2.6 Linear Diophantine equations 3.1 Congruence mod n 3.2 Congruence classes and their arithmetic	2.25 hours pre-recorded video	Elements of Number Theory Chapter 2 & 3
Day 4	3.3 Inverses mod p 3.4 Fermat's little theorem 3.5 Congruence theorems of Wilson and Lagrange 3.6 Inverses mod k	1.5 hours Zoom meeting 0.75 hours pre-recorded video	Elements of Number Theory Chapter 3
Day 5	3.7 Quadratic Diophantine equations 4.1 Trapdoor functions 4.2 Ingredients of RSA 4.3 Exponentiation mod n	2.25 hours pre-recorded video	Elements of Number Theory Chapter 3 & 4
Day 6	4.4 RSA encryption and decryption 4.5 Digital signatures 4.6 Other computational issues 6.1 $\mathbb{Z}[i]$ and its norm 6.2 Divisibility and primes in $\mathbb{Z}[i]$ and $\mathbb{Z}$	2.25 hours pre-recorded video	Elements of Number Theory Chapter 4
Day 7	6.3 Conjugates 6.4 Division in $\mathbb{Z}[i]$ 6.5 Fermat's two square theorem 6.6 Pythagorean triples 6.8 Discussion	1.5 hours Zoom meeting 0.75 hours pre-recorded video	Elements of Number Theory Chapter 6
Day 8	7.1 The equation $y^3=x^2+2$ 7.2 The division property in $\mathbb{Z}[\sqrt{-2}]$ 7.3 The gcd in $\mathbb{Z}[\sqrt{-2}]$ 7.4 $\mathbb{Z}[\sqrt{-3}]$ and $\mathbb{Z}[\zeta_3]$	2.25 hours pre-recorded video	Elements of Number Theory Chapter 7 Assignment 1 due
Day 9	9.1 Primes $x^2+y^2$ , $x^2+2y^2$ , and $x^2+3y^2$ 9.2 Statement of quadratic reciprocity 9.3 Euler's criterion 9.4 The value of $\left(\frac{2}{q}\right)$	1.5 hours Zoom meeting 0.75 hours pre-recorded video	Elements of Number Theory Chapter 9



Day 10	9.6 The Chinese remainder theorem 9.7 The full Chinese remainder theorem & Exam Review	2.25 hours pre-recorded video	Elements of Number Theory Chapter 9
Day 11	Chapter 1 Why Abstract Algebra? Chapter 2 Operations	2.25 hours pre-recorded video	A book of Abstract Algebra
Day 12	<b>Exam 1</b>	2.25 hours online exam proctored under zoom- meeting camera.	
Day 13	Chapter 3 The Definition of Groups Chapter 4 Elementary Properties of Groups	2.25 hours pre-recorded video	A book of Abstract Algebra
Day 14	Chapter 5 Subgroups	1.5 hours Zoom meeting 0.75 hours pre-recorded video	A book of Abstract Algebra
Day 15	Chapter 6 Functions	2.25 hours pre-recorded video	A book of Abstract Algebra Assignment 2 due
Day 16	Chapter 7 Groups of Permutations	2.25 hours pre-recorded video	A book of Abstract Algebra
Day 17	Chapter 8 Permutations of a Finite Set & Exam Review	1.5 hours Zoom meeting 0.75 hours pre-recorded video	A book of Abstract Algebra
Day 18	<b>Exam 2</b>	2.25 hours online exam proctored under zoom- meeting camera.	A book of Abstract Algebra Assignment 3 due