Shanghai Jiao Tong University

MA328 Real Analysis

Instructor Information:	TBD		
Term:	December 16, 2019 - January 7, 2020	**Credits:**	4 units
Class Hours:	Monday through Friday, 160 mins per teaching day		
Discussion Sessions:	2 hours each week, conducted by teaching assistant(s)		
Total Contact Hours:	64 contact hours (1 contact hour = 45 mins, 2880 mins in total)		
Required Texts (with ISBN):	N/A		
Prerequisite:	Students are expected to pass one of Calculus 2, Linear Algebra, Accelerated Mathematics 1		

Course Overview

This subject introduces the field of mathematical analysis both with a careful theoretical framework as well as selected applications. Many of the important results are proved rigorously and students are introduced to methods of proof such as mathematical induction and proof by contradiction.
The important distinction between the real numbers and the rational numbers is emphasized and used to motivate rigorous notions of convergence and divergence of sequences, including the Cauchy criterion. These ideas are extended to cover the theory of infinite series, including common tests for convergence and divergence. A similar treatment of continuity and differentiability of functions of a single variable leads to applications such as the Mean Value Theorem and Taylor's theorem. The definitions and properties of the Riemann integral allow rigorous proof of the Fundamental Theorem of Calculus. The convergence properties of sequences and series are explored, with applications to power series representations of elementary functions and their generation by Taylor series. Fourier series are introduced as a way to represent periodic functions.

Learning Outcomes

On completion of this subject students should

1. Acquire an appreciation of rigour in mathematics, be able to use proof by induction, proof by contradiction, and to use epsilon-delta proofs both as a theoretical tool and a tool of approximation;
2. Understand the theory and applications of the Riemann integral and improper integrals; Be able to determine the convergence and divergence of infinite series;
3. Have a good knowledge of the theory and practice of power series expansions and Taylor polynomial approximations;
4. Understand the role of Fourier series in representing periodic functions.
5. Problem-solving skills: the ability to engage with unfamiliar problems and identify relevant solution strategies;
6. Analytical skills: the ability to construct and express logical arguments and to work in abstract or general terms to increase the clarity and efficiency of analysis;
7. Collaborative skills: the ability to work in a team;
8. Time management skills: the ability to meet regular deadlines while balancing competing commitments.

Grading Policy

Assignments	20%
Midterm Test	40%
Final Examination	40%

Grading Scale

Number grade	Letter grade	GPA
90-100	A	4.0
85-89	A-	3.7
80-84	B+	3.3
75-79	B	3.0
70-74	B-	2.7
67-69	C+	2.3
65-66	C	2.0
62-64	C-	1.7
60-61	D	1.0
≤59	F (Failure)	0

Class Schedule

Day 1	Propositional Logic: Connectives, Truth Table, Logical Equivalence
Day 2	First Order Logic: Substitution, Quantifiers; Statement Proofs: Universal, Existential; Tautology, Contradiction, Converse, Contrapositive
Day 3	Proof: Conditional and Biconditional Statements; Set Theory; The Structure of Proofs; Assignment 1 Due
Day 4	Bounded Sets: Ordered Set, the Least Upper Bound Property; Fields: the Axioms for a Field, Dedekind Cuts
Day 5	Proof: Indirect Proofs, Contrapositive Proofs, Proof by Contradiction, Proof by Induction
Day 6	Absolute Value Function and Inequalities; The Archimedean Property, Density of the Rationals; Assignment 2 Due
Day 7	**Midterm Exam**
Day 8	Functions and Sequences: Epsilon-M Definition, Bounded Sequences, Cauchy Criterion
Day 9	Limits: Algebra of Limits, Limit Points, Continuous Functions, Epsilon- Delta Definition, One-Sided Limits, Left and Right Limits, Limit Laws
Day 10	Continuity: Bounded and Monotonic Functions, Intermediate Value Theorem, Open and Closed Subset, Compact Subsets; Assignment 3 Due
Day 11	Differentiability: Rolle's Theorem, Mean Value Theorem, Taylor's Theorem;
Day 12	Integration: Riemann Integration, Algebraic Properties, Improper Integrals
Day 13	Series: Convergence and Divergence, Common Tests;
Day 14	Series: Taylor Series, Fourier Series; Assignment 4 Due
Day 15	**Final Exam**